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Paper No. 19

ELECTRO-MAGNETIC INDUCTION DAMPING OF VIBRATORY MOTION

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Reprinted from Conference Paper No. CP 60-194,
American Institute of Electrical Engineers;
Dec. 8, 1959.

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Conference Paper

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A paper recommended by the AIEE Aero-Space Transportation Committee for presentation at the AIEE Winter General Meeting, New York, N. Y., January 31-February 5, 1960. Manuscript submitted October 2, 1958; made available for printing December 8, 1959.

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Paper No. CP
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ELECTRO-MAGNETIC INDUCTION DAMPING OF VIBRATORY MOTION

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Synopsis: The problem of damping vibratory motion is frequently encountered in engineering design. Specifically, in the aircraft industry the vibration of skin sections subjected to intense sound levels requires damping in order to prevent rapid fatigue of the metal. Electro-magnetic induction damping affords one effective method of limiting the amplitude of vibration experienced by the skin section thereby reducing the fatigue of the metal. The damping force is achieved by allowing an aluminum ring to vibrate in a strong magnetic field. The ring is fixed to the inner surface of the skin section and the permanent magnet which provides the field is rigidly mounted on the supporting structure. Relative motion between the ring and the magnetic field results in an induced voltage in the ring. Although the voltage is low, the very small electrical resistance of the ring results in the flow of large current. The resulting current reacts with the magnetic field to produce the restraining force on the ring. Experimental results indicate that with this type of damping the vibration amplitude of a typical skin panel under constant vibratory excitation will be only three percent of the amplitude it would have without damping.

Eddy current damping has found many applications in engineering practice. One of its great advantages is that the damping force is a linear function of velocity rather than displacement. This fact suggests its use as a means of suppressing vibratory motion. The application considered in this paper concerns the reduction of the vibration of aircraft skin sections subjected to intense sound levels. The vibratory condition is simulated by mounting test sections of the skin with supporting structure on an electrically driven shaker table. By this method the amplitude and frequency may be controlled and the test conducted at the resonant frequency of the skin section.

Nomenclature

Rationalized meter-kilogram-second units have been presumed.

- e = induced electric potential.
- B = magnetic flux density.
- l = length of the conductor.
- v = velocity of relative motion between the conductor and the magnetic field or particle velocity.
- F = force on a current carrying conductor in a magnetic field.
- i = electric current flowing in the conductor placed in the magnetic field.
- R = electrical resistance of the conductor.
- ρ = electrical resistivity of the conductor.
- a = cross-sectional area of the conductor.
- ϕ = magnetic flux piercing the conductor.
- V = volume of the conductor in the magnetic field.
- y = particle displacement.
- $\frac{dv}{dt}$ = particle acceleration.
- A = maximum amplitude of vibration measured from the neutral position.
- $\omega = 2\pi f$
- $\omega' = 2\pi f'$
- f = frequency of vibration without damping.

f' = frequency of vibration with damping.
m = mass of ring and moving skin section.
F' = Newtonian force.
W = energy of the mass without damping.
W' = energy of the mass with damping.
A' = reduced amplitude of vibration after electro-magnetic induction damping is applied.

$$k = \frac{\text{accoustical energy absorbed without damping}}{\text{accoustical energy absorbed with damping}}$$

Development of Basic Theory

Consider a conducting ring moving parallel to its axis and cutting a radial uniform magnetic field as shown in Figure 1.

For this case the magnetic field, current carrying conductor, and direction of force or motion are mutually perpendicular, which means that the induced voltage may be simply calculated.

$$e = Blv \qquad (1)$$

The force on the current carrying conductor under the same conditions is given by:

$$F = Bli \qquad (2)$$

Considering the ring as a pure resistance:

$$e = iR \quad R = \rho \frac{l}{a} \qquad (3)$$

from which:

$$e = i\rho \frac{l}{a}$$

Equating equations (1) and (3):

$$Blv = i\rho \frac{l}{a}$$

from which:

$$i = \frac{Bla}{l\rho} v = \frac{Ba}{\rho} v \qquad (4)$$

Substituting the second form of equation (4) into equation (2):

$$F = \frac{B^2al}{\rho} v = \frac{B^2V}{\rho} v \qquad (5)$$

Note that from equation (5) the resisting force on the ring is a direct function of the velocity rather than the displacement. This means that for simple harmonic motion, the force is a maximum for zero displacement.

For the case of simple harmonic motion, it is instructive to calculate the energy relationships both with and without the electro-magnetic induction force.

Consider a mass increment, m, executing simple harmonic motion as given by the following relationship:

$$y = A \sin \omega t$$

$$v = \frac{dy}{dt} = \omega A \cos \omega t$$

$$\frac{dv}{dt} = \frac{d^2y}{dt^2} = -\omega^2 A \sin \omega t = -\omega^2 y$$

$$F' = m \frac{dy}{dt} = -m\omega^2 y \qquad (6)$$

The maximum energy of the freely vibrating mass as it passes through the neutral position may be calculated thus:

$$W = \int_{-A}^{0} F' dy = -m\omega^2 \int_{-A}^{0} y dy = -m\omega^2 \left[\frac{y^2}{2} \right]_{-A}^{0}$$

$$W = m\omega^2 \frac{A^2}{2} \qquad (7)$$

Considering the application of the force (equation 5) due to electro-magnetic induction damping, the energy of the vibrating mass as it passes through the neutral position may be calculated thus:

$$W' = \int_{-A}^{0} \left(\frac{B^2 V}{\rho} \omega' A' \cos \omega' t - m\omega'^2 y \right) dy$$

Upon completing the integration, the following relation is obtained:

$$W' = \frac{1}{2} \left[\frac{\pi}{2} \frac{B^2 V}{\rho} \omega' + m\omega'^2 \right] A'^2 \qquad (8)$$

Considering the accoustical energy absorbed to be reduced by a factor of $\frac{1}{k}$ due to the electro-magnetic induction damping, the following expression may be obtained from equations (7) and (8):

$$W = kW'$$

$$\frac{1}{2} m\omega^2 A^2 = \frac{k}{2} \left[\frac{\pi}{2} \frac{B^2 V \omega'}{\rho} + m\omega'^2 \right] A'^2$$

from which it may be shown that:

$$\frac{A'}{A} = \frac{1}{\sqrt{\frac{k \pi B^2 V \omega'}{2 m \rho \omega^2} + k\left(\frac{\omega'}{\omega}\right)^2}} \qquad (9)$$

and,

$$K = \frac{1}{\left(\frac{A'}{A}\right)^2 \left[\frac{\pi B^2 V \omega'}{2m \rho \omega^2} + \left(\frac{\omega'}{\omega}\right)^2 \right]} \qquad (10)$$

A consideration of equation (9) indicates that electro-magnetic induction will be more effective at low vibration frequencies and high flux density.

Experimental Results

In order to determine the damping effectiveness and to evaluate k, an experimental study was made. An aluminum ring three inches in diameter and 0.04 inch thick was mounted in the center of an aluminum skin section.

The skin section was 0.032 inch thick, 8.25 inches wide, and 26 inches long. The ring was mounted in such a manner that 1/2 inch of its width extended into an air gap of a permanent magnet which was supported by the structure holding the skin. The flux density in the air gap was 1.3 weber per square meter. The entire assembly was mounted on a vibrator table and subjected to variable frequency excitation. With the aluminum ring not in the magnetic field, a resonant frequency was found at 72 cycles per second at which a vibrator amplitude of 0.0005 inch resulted in a skin amplitude of 0.040 inch. With the aluminum ring in the magnetic field, a resonant frequency was found at 89 cycles per second at which a vibrator amplitude of 0.0005 inch resulted in a skin amplitude of 0.0012 inch. Hence, the amplitude of skin vibration with electro-magnetic induction damping is only three percent of its value without the damping.

On the basis of this test, equation (10) may be used to evaluate k, thus:

$$V = (0.04 \times 0.5 \times 3 \times 2.54^3 \times 10^{-6}) = 3.08 \times 10^{-6} \text{ Cubic Meter}$$

$$\rho = 2.83 \times 10^{-8} \text{ Ohm-Meter For Aluminum}$$

$$k = \frac{1}{0.03^2 \left[\frac{\pi \times 1.3^2 \times 3.08 \times 10^{-6} \times 2\pi \times 89}{2m \times 2.83 \times 10^{-8} \times (2\pi \times 72)^2} + \left(\frac{89}{72}\right)^2 \right]} = \frac{1}{9 \times 10^{-4} \left(\frac{0.79}{m} + 1.53\right)}$$

By assuming $m = 84$ grams $= 0.084$ kilograms, a value of k may be found:

$$k = \frac{1}{9 \times 10^{-4}\ (10.93)} = \frac{1}{0.984 \times 10^{-2}} = 102.$$

This indicates that only one percent of the energy absorbed without damping is absorbed with damping.

By using equations (4) and (5), the ring current and force may be calculated. From equation (4), the maximum value of the ring current under the conditions of this test may be calculated to be 10.2 amperes, and from equation (5), the maximum value of the force may be calculated to be 0.71 pound. It is interesting to note that had the amplitude of vibration been 0.040 inch, the maximum current would have been 338 amperes, and the maximum force would have been 23.6 pounds.

With the aluminum ring in the magnetic field the second resonant frequency was found to be 177 cycles per second at which a vibration amplitude of 0.0005 inch resulted in a skin amplitude of 0.0011 inch. Thus at this second resonant frequency the amplitude ratio is seven in the absence of the magnetic field as compared to 2.2 when electro-magnetic induction damping is used. Using this data in equation (10) the value of k at 177 cycles per second is found to be 2.45. This means that at this second resonant frequency damping reduced the vibration amplitude to 31 percent of its undamped value due to the fact that only 41 percent as much energy was absorbed.

It is interesting to note that in the absence of damping the amplitude ratio at the low resonant frequency was 33.3 times as large as that at the high resonant frequency; whereas, with damping it was only 3.18 times as large. In fact, with damping the amplitude ratio at the low resonant frequency is only 9.1 percent larger than that at the high resonant frequency. This may be interpreted to mean that with electro-magnetic induction damping the skin section is rendered essentially non-resonant. This is further substantuated by the fact that although the frequency was varied continuously from 20 cycles per second to 500 cycles per second no other frequency was found at which a significant resonant effect could be observed. Scanning this same frequency range without damping revealed no other resonant frequency at which the amplitude ratio was as much as 1.3.

Tests made with the ring clamped to prevent relative motion between it and the magnet revealed several other resonant frequencies. The amplitude ratios at these frequencies were much larger than those experienced when induction damping was used. This statement is based on audible indication and upon visual observation made by using a strobotac light. Incidentally, the resonant frequencies were quite easily determined in this test, whereas with induction damping the two resonant frequencies were much less sharp and consequently more difficult to determine audibly.

Figures 2 and 3 show the MB-Model C-5DHM Shaker used and the skin section in position. The velocity type probe mounted above the skin section is a MB-115 model. The pick-up coil shown taped to the top of the magnet in Figure 2 was used to check resonance. The alternating current which flowed in the ring due to the voltage induced by its motion served, in turn, to induce a voltage in the pick-up coil. The magnitude and frequency of this voltage as observed on a cathode ray oscilliscope was indicative of the motion of the skin section.

CONCLUSIONS

Electro-magnetic induction damping may be effectively used to reduce the amplitude of low frequency vibration. Experimental results indicate that vibration amplitudes may be reduced by as much as 97 percent. This reduction may be attributed to a 99 percent decrease in the kinetic energy of the vibrating mass. The I^2R loss in the ring dissipates the energy which would otherwise tend to fatigue the rivets holding the skin section to the structure. By using the new ceramic magnets induction damping may be used on larger skin pannels with no greater increase in weight than would result from structural changes which might be used to solve the problem. Structural changes which reduce vibration at a given resonant frequency usually result in the creation of new resonant frequencies whereas this is not the case when induction damping is used. This type of damping may be used in any situation in which a low frequency high intensity vibration is encountered. In addition to suppressing the vibration of aircraft skin sections it might be used to reduce resonant vibration excited by power frequencies. A further suggested application is the suppression of vibration in various rotating machine structures which may have low resonant frequencies.

The author acknowledges the help and encouragement of W. G. Nassimbene and P. F. Elliott of the Douglas Aircraft Company, Incorporated.

REFERENCES

1. Robey, D. H., "Magnetic Dispersion of Sound in Electrically Conducting Plates," Journal of the Acoustical Society of America, Volume 25, Number 4, pp. 603-609, July 1953.

2. Olson, H. F., "Electronic Control of Noise, Vibration, and Reverberation," Journal of the Acoustical Society of America, Volume 28, Number 5, pp. 966-972, September 1956.

3. Lassiter, L. W., Hess, R. W., and Hubbard, H. H., "An Experimental Study of the Response of Simple Panels to Intense Acoustic Loading," Journal of the Aeronautical Sciences, Volume 24, Number 1, pp. 19-24, January 1957.

4. Douglas Aircraft Company, Incorporated, "Electromagnetic Induction Damping," Engineering Report No. LB-25726.

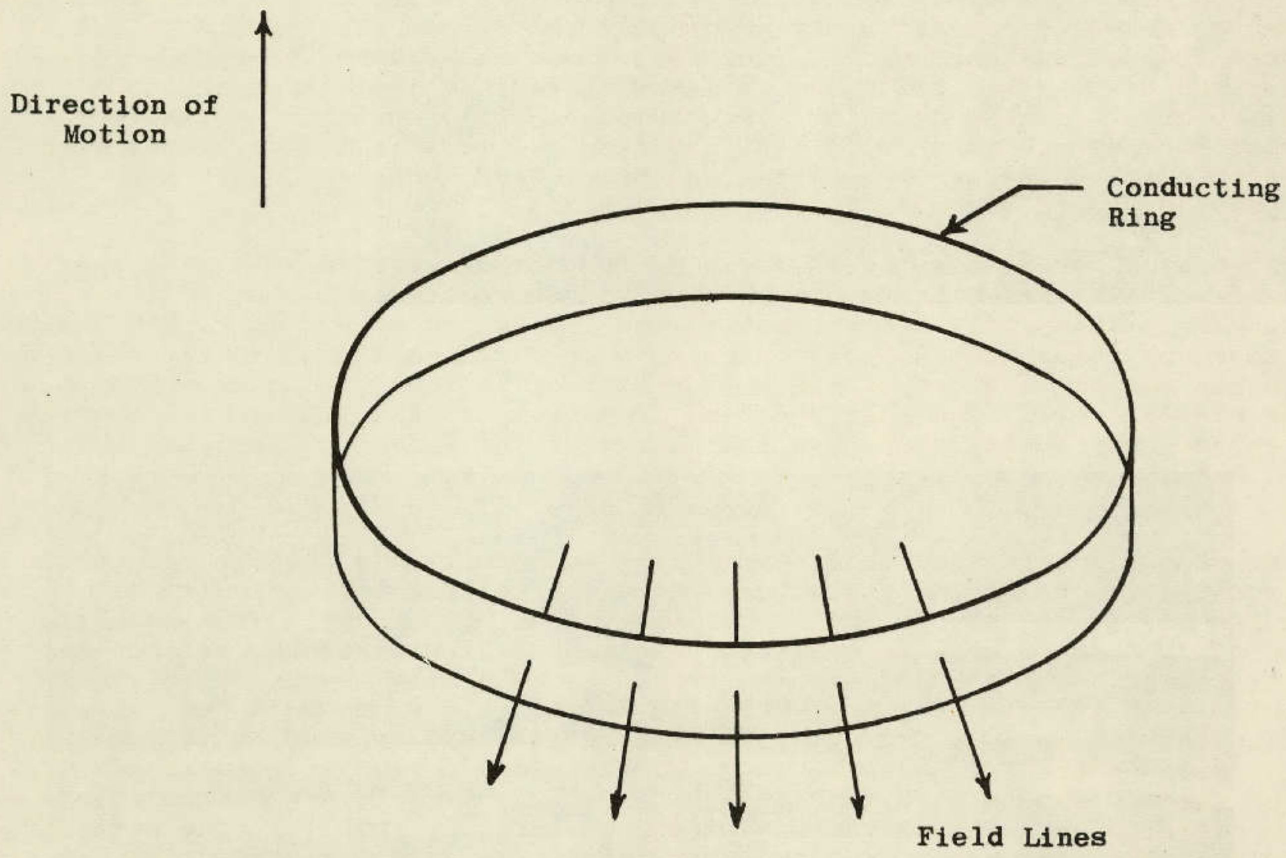

Fig. 1. Geometric Relation of Ring, Magnetic Field, and Motion

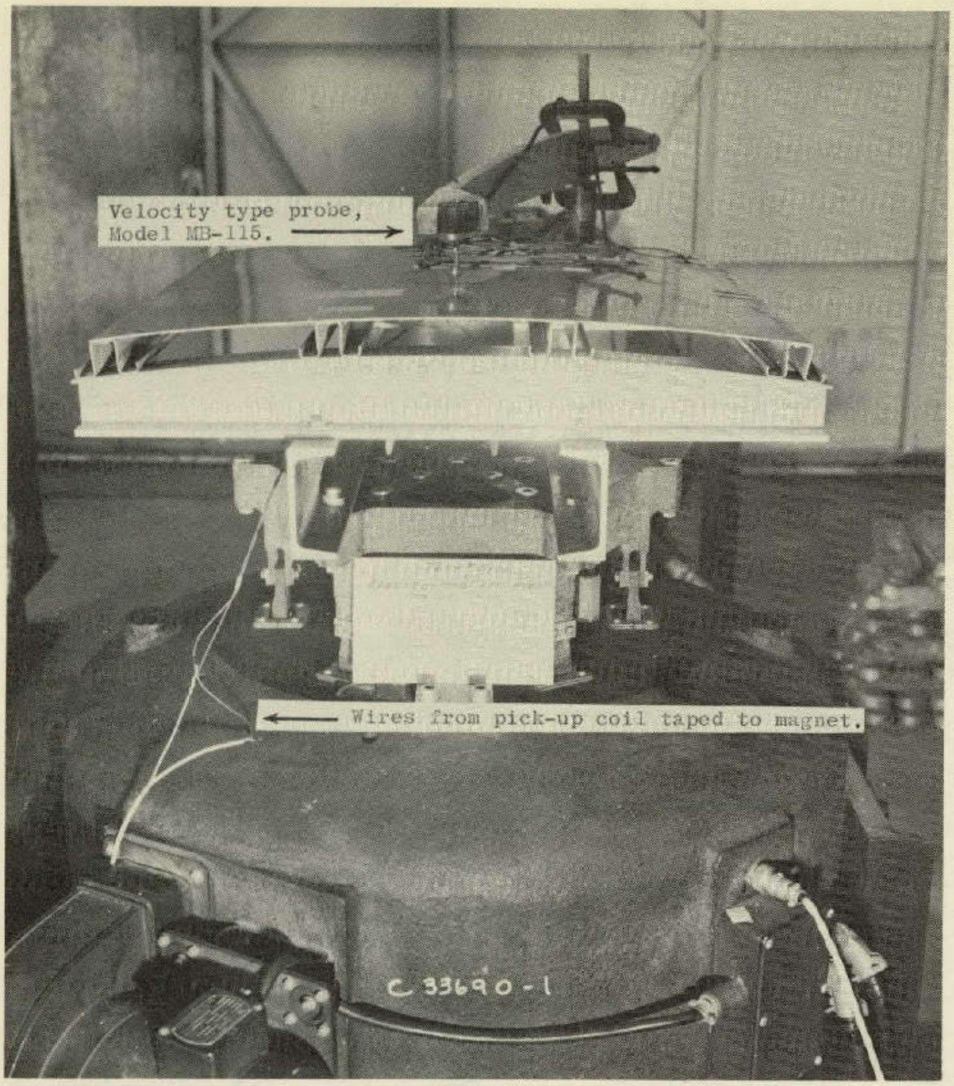

Fig. 2. (Left) Aircraft Skin Section Mounted On The MB-Model-C-5DHM Shaker.

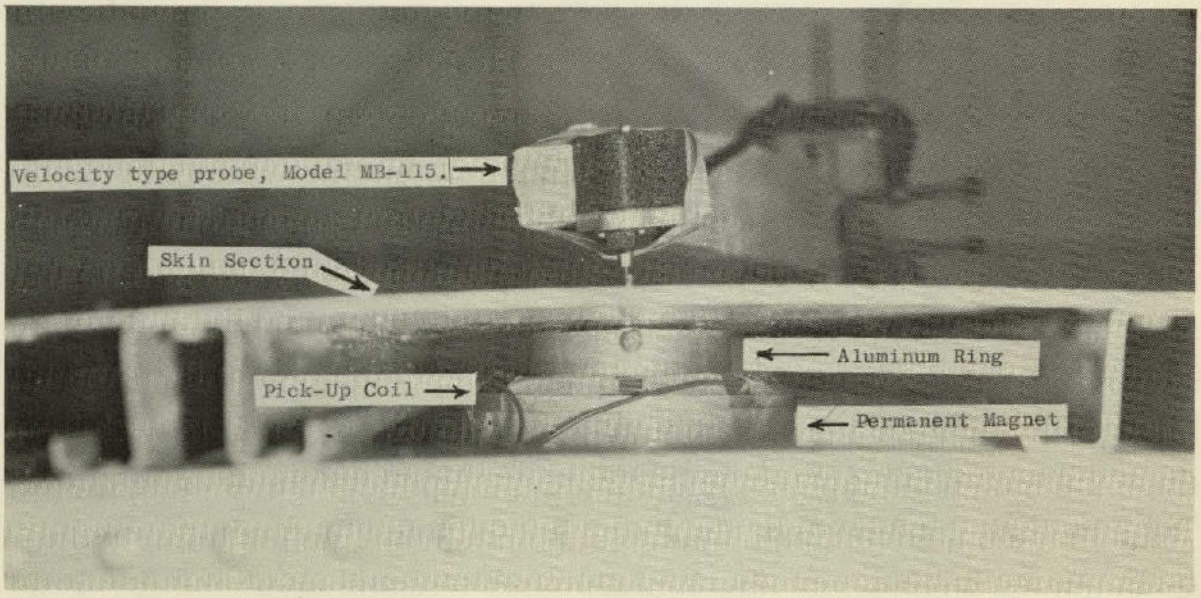

Fig. 3. Aircraft Skin Section With Ring, Magnet, And Velocity Probe, MB-115 In Position.