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Programing the Analog Computer

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Programming the Analog Computer

Here are ways to resolve engineering problems into mathematical equations that can be handled by the d-c analog computer.

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ELECTRONIC ANALOG computers are relatively simple devices for the solution of many applied mathematical problems. Such instruments have found extremely wide applications in design, simulation, and control in various phases of engineering.

Among all types of analog computers, the d-c analog computers are comparatively cheap and easy to operate. With appropriate auxiliary elements, the analog computers can be used to solve problems involving linear and non-linear differential equations, boundary value problems, and repetitive computation of variable functions.

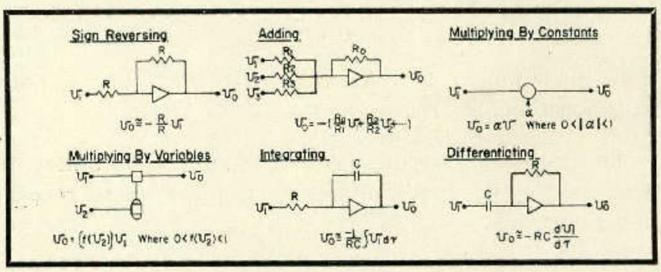
An inherent limitation of analog computer, however, is its inability to handle a large number of variables and operations. Commercial application will therefore be justified by the nature and flexibility of problems, accuracy required, volume of data, and convenience of operations.

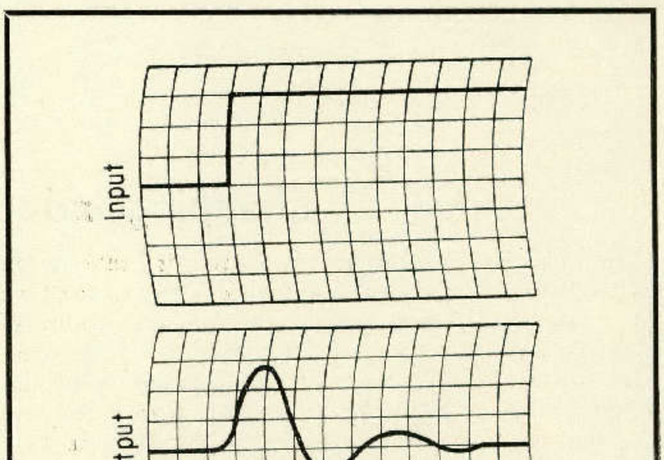
The d-c analog computer is composed of a number of d-c amplifiers and feedback networks. With the addition of time motor, potentiometer or servo multiplier, a d-c analog computer can generally perform many useful mathematical operations. After a network is set up to represent a problem, the boundary values or initial conditions of the problem are fed to the computer as feed voltages. The circuit is closed then and its output voltage varies under the prescribed conditions of the operational machine equation. The computer may be stopped at any time by opening the circuit.

The output voltage of the computer is recorded continuously as the graphical solution of the problem. The change of output voltage may be recorded with respect to the operating time of a computer as shown in Figure 1. Then the output voltage will represent the dependent variable of a problem, and the computer time the independent variable. If a problem involves more than two dependent variables, two of the variables may be recorded by a continuous drum recorder or x-y recorder to yield a regular rectangular graphical plot. The computer solution usually provides 0.5 to 5 percent accuracy.

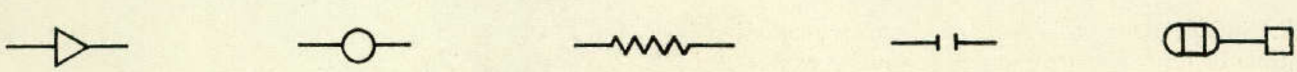
The principal work in connection with the analog computer is the programming work. This includes the mathematical analysis of the problem, transformation of the mathematical expression into an operational equation, and circuit analysis. The physical setup of analog circuits and operation of the computer is more or less mechanical in a sense.

The following problems are presented to demonstrate the general principles involved in programming an analog computer. There are a few methods for setting up machine equations for the analog computer. The examples represent some typical engineering problems and will show the relation of the mathematical equations to the





It's easy to resolve engineering problems into computer setups using these simple symbols



After you decide the best mathematical form which represents the problem, the d-c analog computer can take over and do the calculations.

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components of the analog computer which will solve these equations.

SYMBOLS USED

A Surface area for heat transfer
B Slab thickness
b Conversion factor
C Electrical capacitance
C_p Specific heat capacity
D Diameter
F Moles of feed
f Friction of drag
G Flow rate
g Gravitational constant
v_h Horizontal velocity
K Vapor-liquid equilibrium constant
k Thermal conductivity
L Moles of liquid
l Reactor length
m Positive integer
N Number of theoretical trays
n Moles of component present
P Pressure
p Perimeter
R Electrical resistance
r Reaction rate constant
s Distance from free end
S Function of s
T Temperature of heating medium
t Temperature of system
U Over-all heat transfer coefficient
V Moles of vapor
V_r Volume of reactor
v Vertical velocity
W Weight of system
x Mole fraction in liquid phase
y Mole fraction in vapor phase
z Percent conversion
α (Refer to Discussion)
β (Refer to Discussion)
Θ Function of θ
θ Time of process change
π 3.14
ρ Density
τ Operating time of computer

Linear First-Order Differential Equation

The reaction of A to form B, which in turn reacts to form C, may be represented as follows:

$$A \xrightarrow{r_1} B \xrightarrow{r_2} C$$

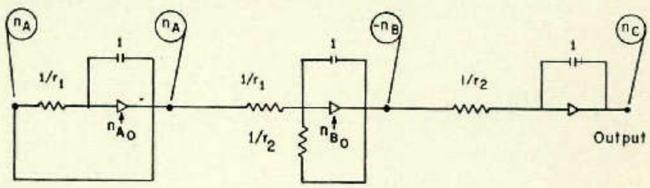
Find the changes in concentrations of A, B, and C with reaction time.

Solution—The reaction rate for such reactions may be expressed by the following equations, assuming no reverse reactions:

$$-\frac{dn_A}{d\theta} = r_1 n_A$$

$$\frac{dn_B}{d\theta} = r_1 n_A - r_2 n_B$$

$$\frac{dn_C}{d\theta} = r_2 n_B$$



An electrical network representing these equations is shown. If the initial concentrations n_{A_0} and n_{B_0} are fed to the computer as voltage equivalents, the circuit will yield n_A, n_B, and n_C in volts as functions of the computer time. If the reaction time and the computer time are expressed in the same unit of seconds, the recorder voltage outputs of n_A, n_B, and n_C will then represent the corresponding concentration changes versus reaction time.

Changing Independent Variables

In a chemical process, a fluid mixture is heated in an agitated tank as shown. The mixture is fed to the tank at a rate of G pounds per hour and a temperature of t_o F. Steam at a constant temperature of T F. is available for heating. The tank initially contains W pounds of fluid mixture at t_o when the steam is turned on. Find the temperature of the mixture leaving the tank as a function of time before steady state is reached.

Solution—In order to simplify the heat balance, assume that the fluid mixture in the tank is at a uniform temperature equal to that of the mixture leaving the tank. A heat balance for the unsteady state will be represented by the following equation:

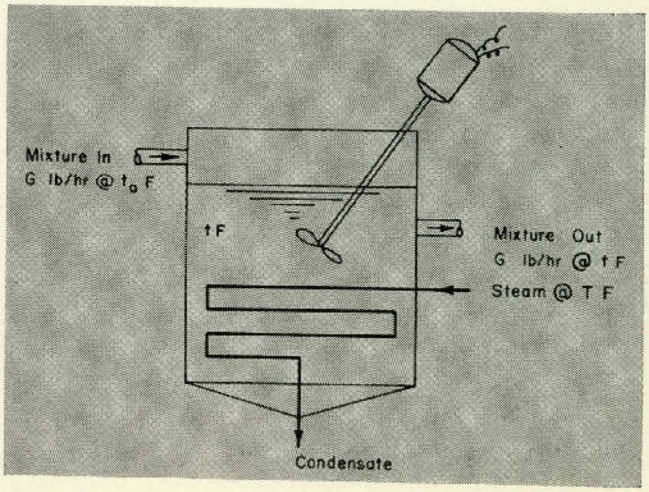
$$UA(T - t) = GC_p(t - t_o) + WC_p\frac{dt}{d\theta}$$

The independent variable Θ of the above equation is the time of unsteady heating and is usually expressed in hours. The equation can be processed through a computer by changing Θ into the computer time τ in seconds by the following relationship:

$$\tau = 3600\,\Theta$$

Hence $$\frac{dt}{d\tau} = \frac{UAT - GC_pt_o}{3600\ WC_p} - \frac{UA + GC_p}{3600\ WC_p}t$$

Since U, A, T, G, C_p, t_o, and W are constants, the above



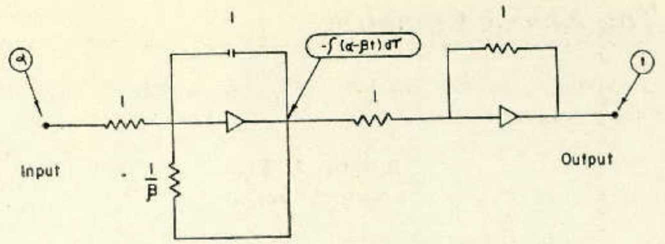
equation can be greatly simplified by writing in the following form.

$$\frac{dt}{d\tau} = \alpha - \beta t$$

where
$$\alpha = \frac{UAT - GC_p t_o}{3600\ WC_p}$$

$$\beta = \frac{UA + GC_p}{3600\ WC_p}$$

The resulting equation is an ordinary differential equation of the first order. It can be solved readily by the computer setup shown here. The network involves one summation and an integration as indicated by the operational equation. The machine output voltage will represent the temperature t of the fluid mixture leaving the tank. The temperature is recorded as a function of machine time τ.



The electronic computation of this problem is rather simple, judging from the fact that only two d-c amplifier units are employed in the circuit. The formal mathematical solution of the equation, however, is much more involved and may be written as follows.

$$t = \frac{\frac{UA}{GC_p}(T - t_o)}{\frac{UA}{GC_p} + 1}\left\{1 - \exp\left[-\left(\frac{UA}{GC_p} + 1\right)\left(\frac{G\theta}{W}\right)\right]\right\} + t_o$$

Linear Second-Order Differential Equation

A metal rod of circular cross section is placed in a stream of hot gas having a constant temperature T. Assume that (1) the temperature of gas surrounding the rod is uniform and (2) the temperature gradient in the radial direction in the rod is negligible. Calculate the temperature distribution within the rod along the axis as a result of simultaneous convection and conduction.

Solution—A heat balance for an element of length ds of the rod may be set up as the following.

Heat in at s by conduction $= -kA\frac{dt}{ds}$

Heat in to the whole section ds by convection around the periphery $= U(T - t)\ pds$

Heat out at s + ds by conduction

$$= kA\left[-\frac{dt}{ds} + \frac{d}{ds}\left(\frac{dt}{ds}\right)ds\right]$$

Equating the total heat input to output, we have

$$-kA\frac{dt}{ds} + U(T - t)\ pds = kA\left[-\frac{dt}{ds} + \frac{d^2t}{ds^2}ds\right]$$

This can be simplified to be

$$\frac{d^2t}{ds^2} = \frac{Up}{kA}(T - t)$$

To apply electronic computation, the temperature t in the rod will again be represented by a voltage. The distance s will be transformed into computer time by letting

$$s = \alpha\tau$$

where α is a constant.

Substituting $\alpha\tau$ for s, the equation becomes

$$\frac{1}{\alpha^2}\frac{d^2t}{d\tau^2} = \frac{Up}{kA}(T - t)$$

In order to simplify the resistance elements in the computer circuit, the value of α may be chosen so that

$$\alpha = \sqrt{\frac{kA}{Up}}$$

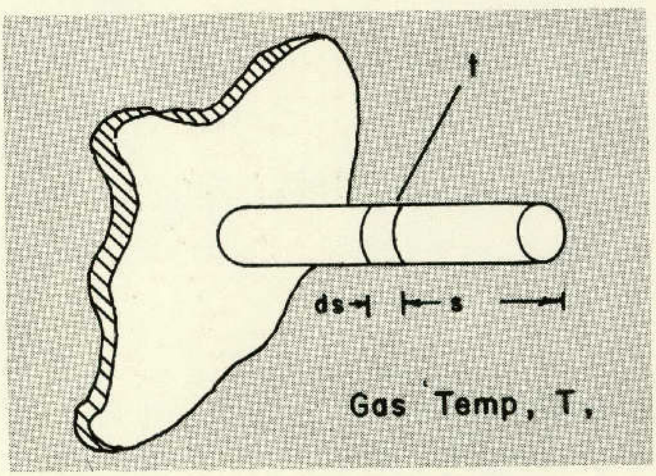
The equation then reduces to

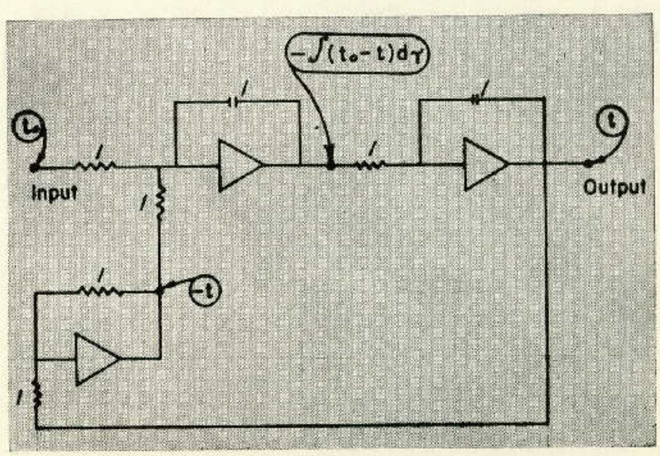
$$\frac{d^2t}{d\tau^2} = T - t$$

where the boundary condition is $t = T$ at $s = 0$ and all values of τ. Written in an integral form, the equation becomes

$$t = \int_o^\tau \int_o^\tau (T - t)\, d\tau d\tau$$

This problem represents a linear differential equation of the second order. The computer setup is prepared according to operational equation shown above. The boundary condition is satisfied by feeding a voltage corresponding to T into the computer. The solution will



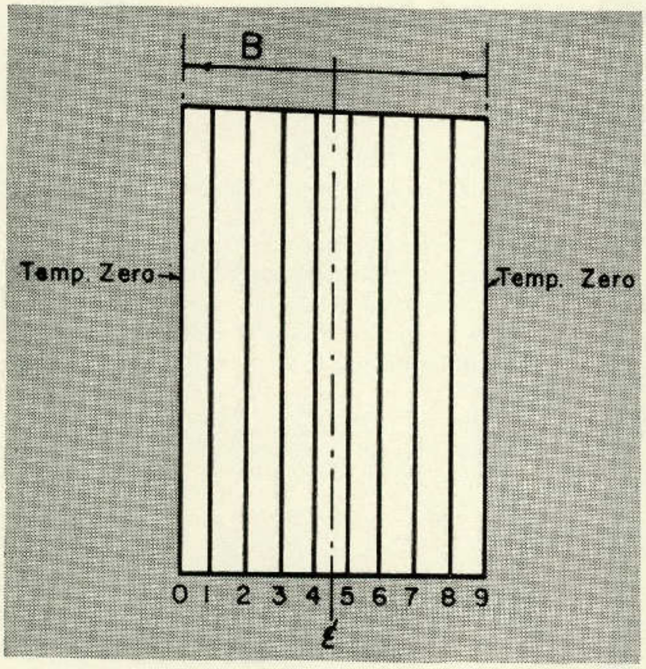


be recorded as temperature t versus computer time τ. Values of τ, of course, may be readily converted to distance s by the following relationship:

$$s = \alpha\tau = \tau\sqrt{\frac{kA}{Up}}$$

Partial Differential Equation

Calculate the temperature distribution in a slab of finite thickness B. The slab is initially at a uniform temperature t′ and its faces are kept at temperature zero.



Solution—Assume that the temperature in the slab varies with distance from the faces only. The unsteady heat transfer problem can then be expressed by the following equation:

$$\frac{\partial t}{\partial \theta} = \frac{k}{C_p\rho}\ \frac{\partial^2 t}{\partial s^2}\quad (0 < s < B, \theta > 0)$$

The boundary conditions are, for $t = t(s, \theta)$,

$$t(s, o) = t' \qquad (o<s<B)$$
$$t(o, \theta) = 0 \qquad (\theta>o)$$
$$t(B, \theta) = 0$$

To separate the variables, assume that

$$t = S(s)\ \Theta(\theta)$$

where $S(s)$ and $\Theta(\theta)$ are functions of s and θ alone. Differentiating the above equation according to the heat transfer equation, we obtain

$$S\Theta' = \frac{k}{C_p\rho} S''\Theta$$

Rearrange the above equation

$$\frac{C_p\rho}{k}\ \frac{\Theta'}{\Theta} = \frac{S''}{S} = \alpha$$

where α is a constant. It follows therefore

$$\frac{d\Theta}{d\theta} = \frac{\alpha k}{C_p\rho}\ \Theta$$

Let

$$\tau = 3600\,\Theta. \quad \beta = \frac{1}{3600}\ \frac{\alpha k}{C_p\rho}$$

Thus,

$$\frac{d\Theta}{d\tau} = \beta\Theta$$

$$\frac{d^2S}{ds^2} = \alpha$$

Where the boundary conditions become

$$\Theta(o) = t'$$
$$S(o) = 0$$
$$S(B) = 0$$

The differential equation of Θ is a function of time and consequently can be solved by a single feedback integration. The equation of S, however, is a function of distance s. The independent variable s should then be transformed into computer time by letting

$$\tau = \frac{b}{B} s$$

where b is a conversion factor in seconds. This provision permits the choice of any value for b corresponding to the slab thickness B. Take for example b = 10 seconds. At 5 seconds of machine time, therefore, the computer output will indicate a function corresponding to s = B/2. the center plane of the slab.

With the change of independent variable, the differential equation of S is transformed into the following form:

$$\frac{b^2}{B^2}\ \frac{d^2S}{d\tau^2} = \alpha S$$

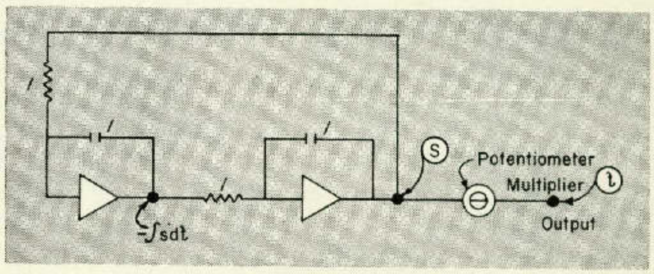
For the sake of simplicity in the computer setup, choose the value of b so that

$$\frac{\alpha B^2}{b^2} = 1$$

The above equation becomes

$$\frac{d^2S}{d\tau^2} = S$$

Here is the computer setup for this equation.



Since the temperature in the slab is determined by two independent variables, i.e., distance s and time θ, a two-dimensional solution may be obtained by correlating temperature against one of the two variables and using the other one as a parameter. According to the derivation, therefore, the computer output of Θ may be multiplied by a constant value of S with the aid of a potentiometer multiplier. The product of $S\Theta$ will indicate the temperature at a certain point in the slab as a function of time. Since both faces of the slab are kept at temperature zero in this problem, a symmetrical temperature distribution in the slab is anticipated. If the slab thickness B is divided into nine even parts, as shown in the figure, only four temperatures at s_1, s_2, s_3, and s_4 will be necessary to express the temperature distribution. A complete set of computer solution would then include four continuous temperature curves of $t(s_1, \theta)$, $t(s_2, \theta)$, $t(s_3, \theta)$ and $t(s_4, \theta)$.

On the other hand, the variable computer output of S may be multiplied by a constant value of Θ in order to generate continuous functions of temperature in the slab in terms of distance at certain instant θ.

The method of finite difference may be used as an alternate solution. The slab is first divided as before into a number of even parts along its thickness. The heat transfer rate may then be approximated in the following.

$$\frac{dt}{ds} \approx \frac{\Delta t}{\Delta s} = \frac{t_{n+1} - t_n}{\Delta s}$$

$$\frac{d^2t}{ds^2} \approx \frac{\Delta^2 t}{\Delta s^2}$$

$$= \frac{1}{\Delta s}\left(\frac{t_{n+1} - t_n}{\Delta s} - \frac{t_n - t_{n-1}}{\Delta s}\right)$$

$$= \frac{1}{\Delta s^2}(t_{n+1} - 2t_n + t_{n-1})$$

where the subscripts $n+1$, n, $n-1$ are referred to the number of cross section in the suab.

Since $\quad \dfrac{\partial t}{\partial \theta} = \dfrac{k}{C_p \rho}\dfrac{\partial^2 t}{\partial s^2}$

We have the following approximation:

$$\frac{\partial t}{\partial \theta} = \frac{k}{C_p \rho}\frac{1}{\Delta s^2}(t_{n+1} - 2t_n + t_{n-1})$$

The increment Δs may be chosen such that

$$\frac{k}{C_p \rho} = \Delta s^2$$

Hence $\quad \dfrac{\partial t}{\partial \theta} = t_{n+1} - 2t_n + t_{n-1}$

The above equation expresses the change of temperature at cross section n in terms of the temperatures at cross sections $n-1$, n, and $n+1$. Solution of this equation is simplified in consideration of the symmetrical temperature distribution in the slab. For example, when $n=9$, we have for all values of θ the following relationships.

$$t_0 = t_9$$
$$t_1 = t_8$$
$$t_2 = t_7$$
$$t_3 = t_6$$
$$t_4 = t_5$$

According to the Schmidt approximation, therefore,

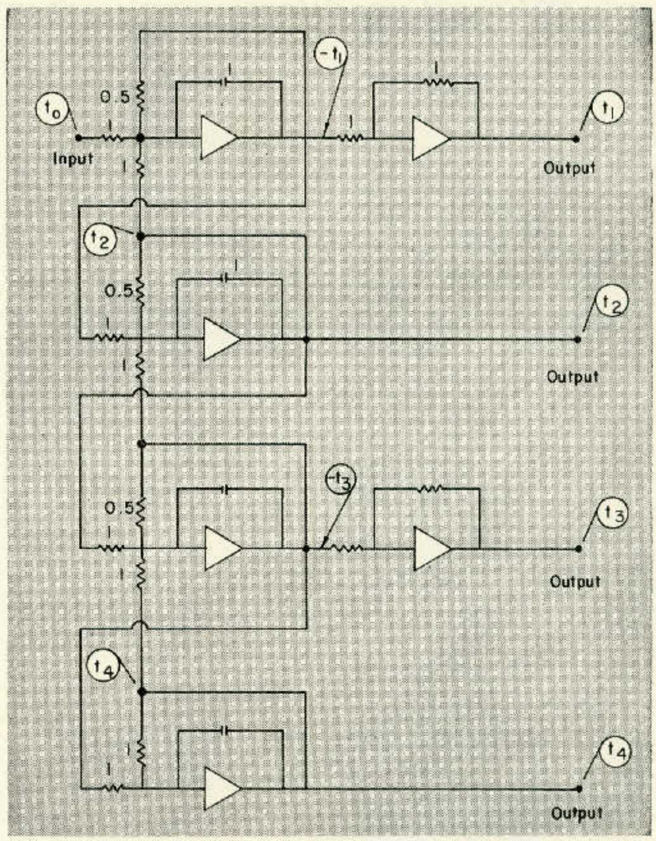
$$\frac{dt_1}{d\theta} = t_0 - 2t_1 + t_2$$

$$\frac{dt_2}{d\theta} = t_1 - 2t_2 + t_3$$

$$\frac{dt_3}{d\theta} = t_2 - 2t_3 + t_4$$

$$\frac{dt_4}{d\theta} = t_3 - 2t_4 + t_5 = t_3 - t_4$$

These equations can be solved simultaneously by analog computer very readily with the network shown here.



The initial slab temperature t′ is satisfied by feeding it to the computer circuit. Values of t_1, t_2, t_3 and t_4 will be obtained as continuous functions of machine time.

The method of finite difference has been developed for numerical analysis of unsteady heat transfer in finite slabs. Its accuracy depends on the number of cross sections in the slab taken into consideration. Application of this method on electronic computer has been found very satisfactory.

The partial differential equation of heat transfer as shown above can be formally solved by means of Fourier series. Its solution is:

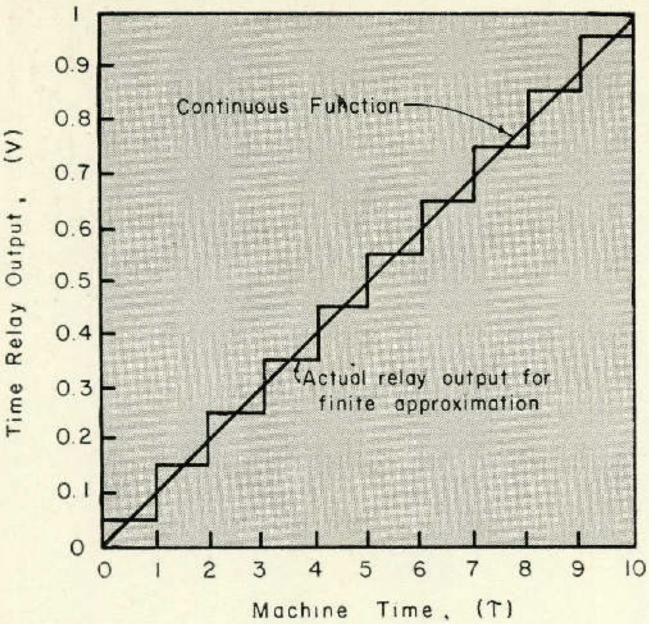
$$t(s, \theta) = \frac{4t'}{\pi}\sum_{1}^{\infty}\frac{1}{2m-1}\exp\left[\frac{-(2m-1)^2\pi^2 k\theta}{C_p \rho B^2}\right]\sin\frac{(2m-1)\pi S}{B}$$

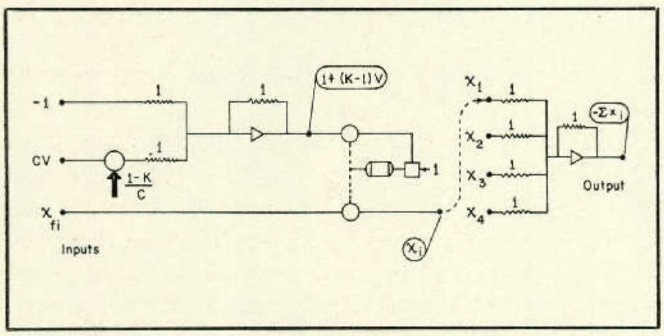
Function Generation by Finite Approximation

Find the ratio of vapor to liquid which exists after flashing a petroleum fraction.

Solution—The flash vaporization of a hydrocarbon mixture is usually expressed by the following equations of material balance:

$$F = L + V$$
$$Fx_{fi} = Lx_i + Vy_i$$





Introducing the equilibrium constant K and taking $F = 1$, these equations may be combined to give the following:

$$x_i = \frac{x_{fi}}{1 + (K_i - 1)V}$$

where $K_i = y_i/x_i$ and V becomes moles of vapor per mole of feed vaporized. Since the summation of mole fraction x of all components in the mixture is 1, we have then

$$\Sigma x_i = \frac{x_{fi}}{1 + (K_i - 1)V} = 1$$

In flash vaporization calculations, it is usually required to determine L and V at predetermined flash temperature and pressure. In the equations, therefore, K_i's are constants at constant temperature and pressure, and V is the independent variable. As has been illustrated in the previous problems, V should be transformed into machine time scale in order to solve the equations by electronic computer. However, to generate the function x_i according to the equation, it is also necessary to feed values of V as voltages into the circuit.

This requirement may be approximated through the additional use of a time relay device. Choose for instance a machine time of 10 seconds to represent the entire range of V (i.e., $0<V<1$). A time relay may then be arranged in accordance with the relation shown in the accompanying graph. The relay output V will change from 0 to 0.5, 1.5, 2.5, etc., in volt every second and will be fed into the circuit shown in the diagram.

This circuit will then generate a function of x_i according to the equation. A summing network will indicate the machine time required for $\Sigma x_i = 1$. If the machine time is 5.64 seconds, for example, V would be 0.564 mole per mole feed at a relay setting of 10 seconds.

A function generating circuit, such as is shown, will be required for each of the components in the feed. With such a circuit setup, it is merely necessary to change the voltage input x_{fi} and equilibrium constant K_i in routine calculations for different feed composition and operating conditions.

A plate-to-plate calculation may be made by applying this method to a multi-component fractionating column. Take the calculation of minimum number of theoretical plates under total reflux for example. We have the following conventional equations:

$$\Sigma x_N = \Sigma \frac{y_N}{K_N} = 1$$

$$y_{N+1} = x_N$$

For computer calculation, the equilibrium constants K may be expressed as functions of temperature alone for different components. The temperature function may be generated by a time motor which is operated in conjunction with the computer. The plate temperature will then be determined by the computer in terms of machine time required to satisfy $\Sigma x_i = 1$. Consecutive plate-to-plate calculation may be carried out by proper network made according to both equations.

The difficulties in applying electronic computer to problems of such nature are the requirement of known functions of equilibrium constant K for all components in the system and a large number of network elements.

Function Generation in a Closed Circuit

It is required to separate an oil-water mixture by means of a standard oil separator shown here. The overflowing level of oil is s feet above the lower end of the oil retention wall. Calculate the minimum length of separation section of the separator.

Solution—The minimum dimension of the separation section may be determined by first estimating the time required for an oil globule to arise from *s* feet beneath the level. To analyze the flow path of an oil globule, its velocity is resolved into vertical component v and horizontal component v_h. The horizontal velocity of an oil globule is constant throughout the separation section. It is equal to the total volume flow rate divided by the cross-sectional area of the separator. The vertical velocity is nevertheless a variable and is dependent upon a force balance which is derived in the following.

Assume an average diameter D for the oil globules.

Accelerational force of the oil globule $= \frac{\pi D^3}{6} \rho_o \frac{dv}{d\theta}$

Buoyancy of the globule $= \frac{\pi D^3}{6} (\rho_w - \rho_o) g$

Drag force against the upflowing globule =

$$\frac{f\left(\frac{\pi D^2}{4}\right)\rho_w v^2}{2}$$

The force balance may then be set up as below:

$$\frac{\pi D^3}{6} \rho_o \frac{dv}{d\theta} = \frac{\pi D^3}{6} (\rho_w - \rho_o) g - \frac{f \pi D^2 \rho_w v^2}{8}$$

$$\frac{dv}{d\theta} = \frac{\rho_w - \rho_o}{\rho_o} g - \frac{3f \rho_w v^2}{4D\rho_o}$$

Or

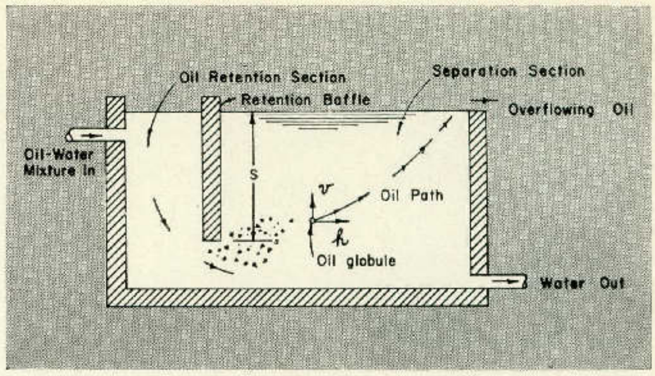
$$\frac{dv}{d\tau} = \alpha - \beta v^2$$

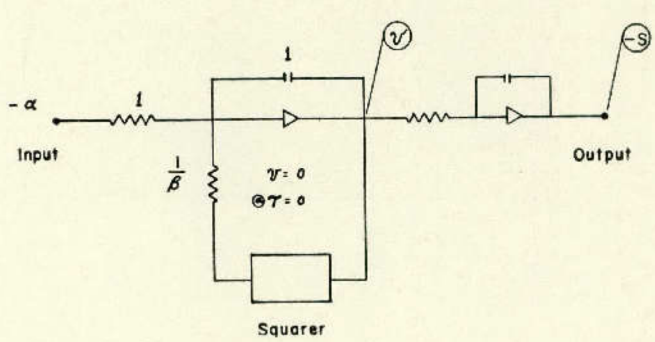
if $\theta = \tau$, $\alpha = \frac{\rho_w - \rho_o}{\rho_o} g$ and $\beta = \frac{3f \rho_w v^2}{4D\rho_o}$. Both α and β are constants provided that a constant value of friction factor f is used.

The vertical velocity v may be integrated with respect to time to give the vertical traverse distance of an oil globule within a given time. Since the total vertical traverse distance s and the function of vertical velocity are known, the above equation is employed to determine the time required for an oil globule to arise to the overflowing surface. An electronic circuit may be set up according to the following equation:

$$S = \int v dt$$
$$= \iint (\alpha - \beta v^2) d\tau d\tau$$

The computer setup as shown here consists of a typical function-generating element in a closed circuit for the variable v.





The circuit output is the vertical traverse path of an oil globule as a function of retention time in the separation section. The minimum retention time for the vertical traverse path equal to the given value s may then be determined. The minimum longitudinal length of the separation section equals simply to the horizontal velocity V_h times the minimum retention time.

Simultaneous Equations

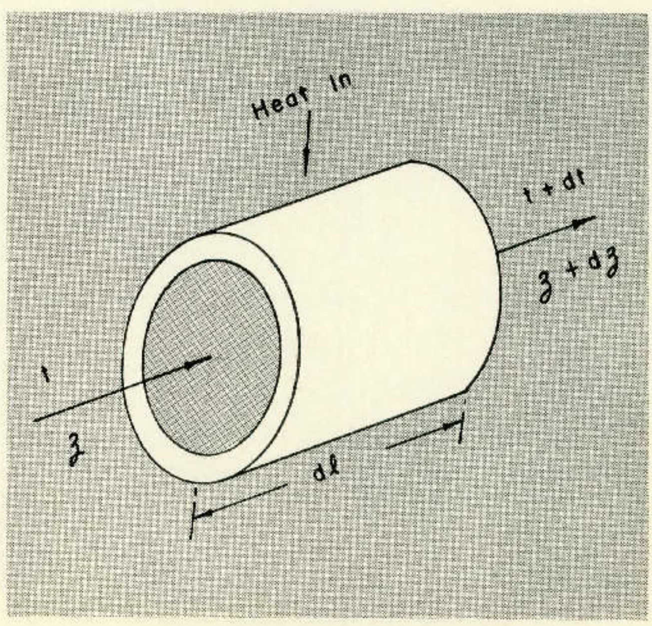
Vapor of sulfuryl chloride at 200 F and 1.2 atm. is fed to a 1½-inch reactor tube at a rate of 418 pounds per hour. The reaction tube is heated at a rate of 5000 Btu/(hr.)(sq. ft.) based on the internal area. The reactor tube has an inside diameter of 1.334 inches. At elevated temperature the SO_2Cl_2 decomposes by a first-order reaction to form SO_2 and Cl_2. It is desired to decompose 98 percent of the SO_2Cl_2 fed. The pressure drop in the reactor and the reverse reaction may be neglected. Calculate the length of reactor tube required.

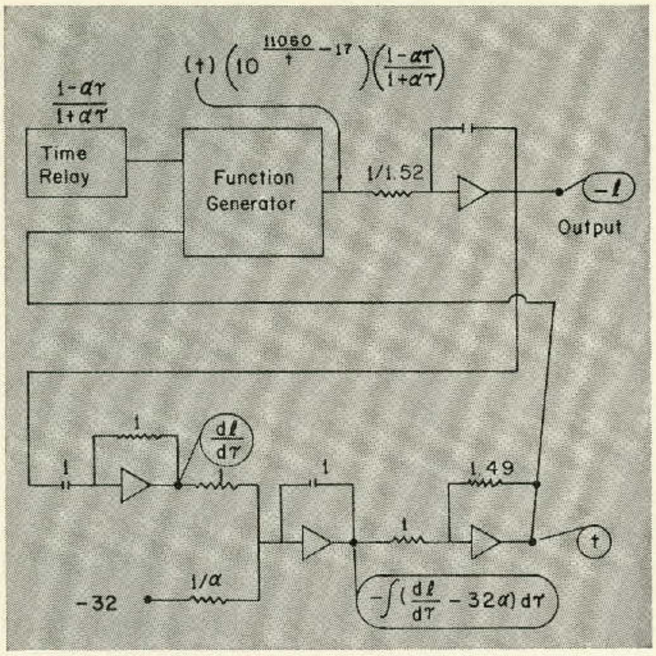
The rate constant r for the uncatalyzed reaction may be represented by the following expression.

$$r = (6.427)\ (10^{15}) e^{-50610/t}\ sec^{-1}.$$

where the temperature t is in °K.

Solution—The decomposition of sulfuryl chloride is an endothermic reaction. Since there is an excess of external heat supply to the reactor, the reaction temperature will change along the entire length of the reactor tube. While the reaction rate is a function of





temperature, the conversion of sulfuryl chloride per unit length of the reactor will not be uniform. Calculation of the reactor length will consequently be made by heat and material balance over a differential section of the reactor.

Assume that in a differential section dl of the reactor there occurs a conversion of dz mol/mol feed and the temperature increases by dt. A heat balance may be established as follows.

Since M.W. of SO_2Cl_2 = 134.91

Feed rate = 418/(134.91)(3600)
= 0.39 gr. mol/sec.

Internal heating surface = 0.3491 sq. ft./ft.

Average heat of reaction = 10,000 cal/mol SO_2Cl_2

Average specific heat = 21 cal/(mol)(°K)

Consequently, for a differential section of the reactor,

External heat input = $\frac{5000}{3600}$ (252) (0.3491)dl Btu/sec

Sensible heat for temperature increase = (0.39) (21)dt Btu/sec

Heat of reaction = (0.39 dz)(10,000) Btu/sec

The heat balance is then expressed by the equation below.

External heat input = sensible heat for temperature increase + heat of reaction

$$\frac{5000}{3600}(252)(0.3491)\,dl = (0.39)(21)\,dt + (0.39\,dz)(10{,}000)$$

$$122\,dl = 8.19\,dt + 3900\,dz$$

Or,

$$dt = 14.9\,dl - 476\,dz$$
$$= 14.9\,(dl - 32\,dz)$$

The material balance is established by means of the following familiar equation for a differential reactor.

$$V_r = \frac{dn}{d\theta} = F dz$$

where the reaction rate $dn/d\theta$ is in mol SO_2Cl_2 decomposed per second per liter. The reactor has a volume of 28.3 liters per linear foot. We can therefore solve for V_r/F as below.

$$\frac{V_r}{F} = \frac{\pi}{4}\,(1.334^2)\,\frac{1}{144}\,(28.3)\,(dl)\,\frac{1}{0.39} = 0.705\ dl$$

An expression for $dn/d\theta$ may be found in terms of partial pressure, which in turn can be expressed by the percentage of decomposition.

$$\frac{dn}{d\theta} = rn$$
$$= rP/Rt$$
$$= r\,(1.2)\,\frac{1-z}{1+z}\,\frac{1}{Rt}$$
$$= (6.427)(10^{15})\,e^{-50610/Rt}(1.2)\,\frac{1-z}{1+z}\,\frac{1}{Rt}$$

Combine the expressions of V_r/F and $dn/d\theta$ now.

$$0.705\ dl = \frac{dz}{(6.427)(10^{15})\,e^{-50610/Rt}(1.2)\frac{1-z}{1+z}\,\frac{1}{Rt}}$$

Upon simplifying, we have

$$dl = (1.52\ t)\ 10^{\frac{11060}{t} - 17}\ \frac{1-z}{1+z}\,dz$$

In order to process through the computer, choose the reactor length l and reaction temperature t as the dependent variables and percent conversion z independent variable. Assume that:

$$z = \alpha\tau$$

Where α is a constant. We have then

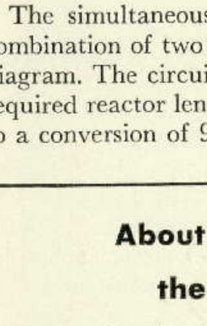
$$l = (1.52\,\alpha)\int t\,10^{\frac{11060}{T} - 17}\,\frac{1-\alpha\tau}{1+\alpha\tau}\,d\tau$$

$$t = (14.9)\int\left(\frac{dl}{d\tau} - 32\,\alpha\right)d\tau$$

where the initial conditions are l = 0 and t = 366.3° K at $\tau = 0$.

The simultaneous equations may be solved with the combination of two circuit systems as shown in the block diagram. The circuit output of l will indicate readily the required reactor length at a computer time corresponding to a conversion of 98 percent. # #

About the Author



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